FOSTERING MATHEMATICAL CREATIVITY THROUGH PATTERNS

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introduction

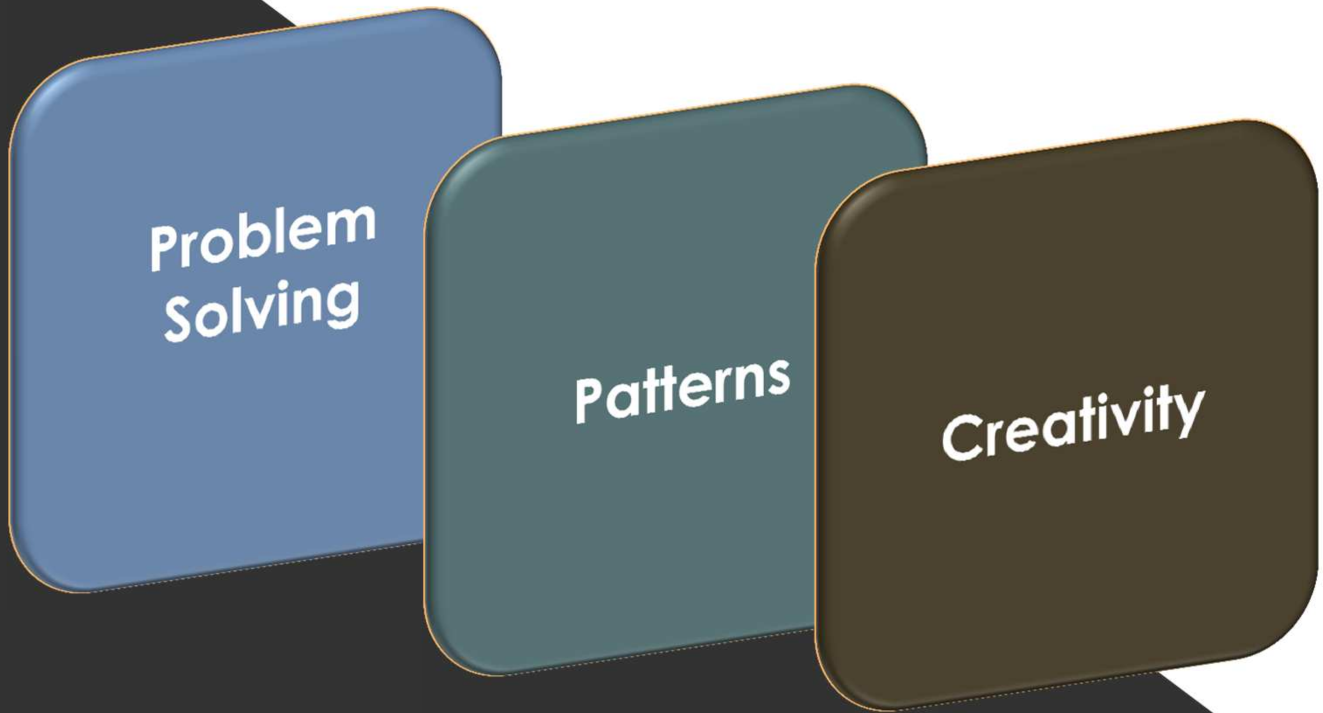
Creativity is a dynamic characteristic that students can develop if teachers provide them appropriate learning opportunities (Leikin, 2009).

- Our recent work in a project about patterns in the teaching and learning of mathematics showed that patterns contribute to the development of mathematical ability and can be a rich context to develop creativity for all students.

- We analyze, through some elementary pre-service teachers classroom episodes, the contribution of pattern tasks to promote creative-solutions by students.

introduction

Teacher Education



Theoretical framework

Learning heavily depends on teachers

One of the major obstacles to reforms is teachers' lack of familiarity with innovative instructional practices and tools
(Heibert et al., 2007)

Teachers **must have a profound understanding of fundamental mathematics**
(Ma, 1999)

Teachers must have **an in-depth understanding of the mathematical thinking of their students to support the development of their mathematical competence**
(Franke et al., 2007)

Teachers must interpret the curriculum and select *good* curricular materials and strategies to use in the classroom in a creative form, and also be mathematically competent to analyze their students solutions.
(Stein & Smith, 2009)

What students learn is largely influenced by the tasks given to them
(Doyle,1988; Hiebert &Wearne,1993; Stein&Smith,1998; Vale,2010).

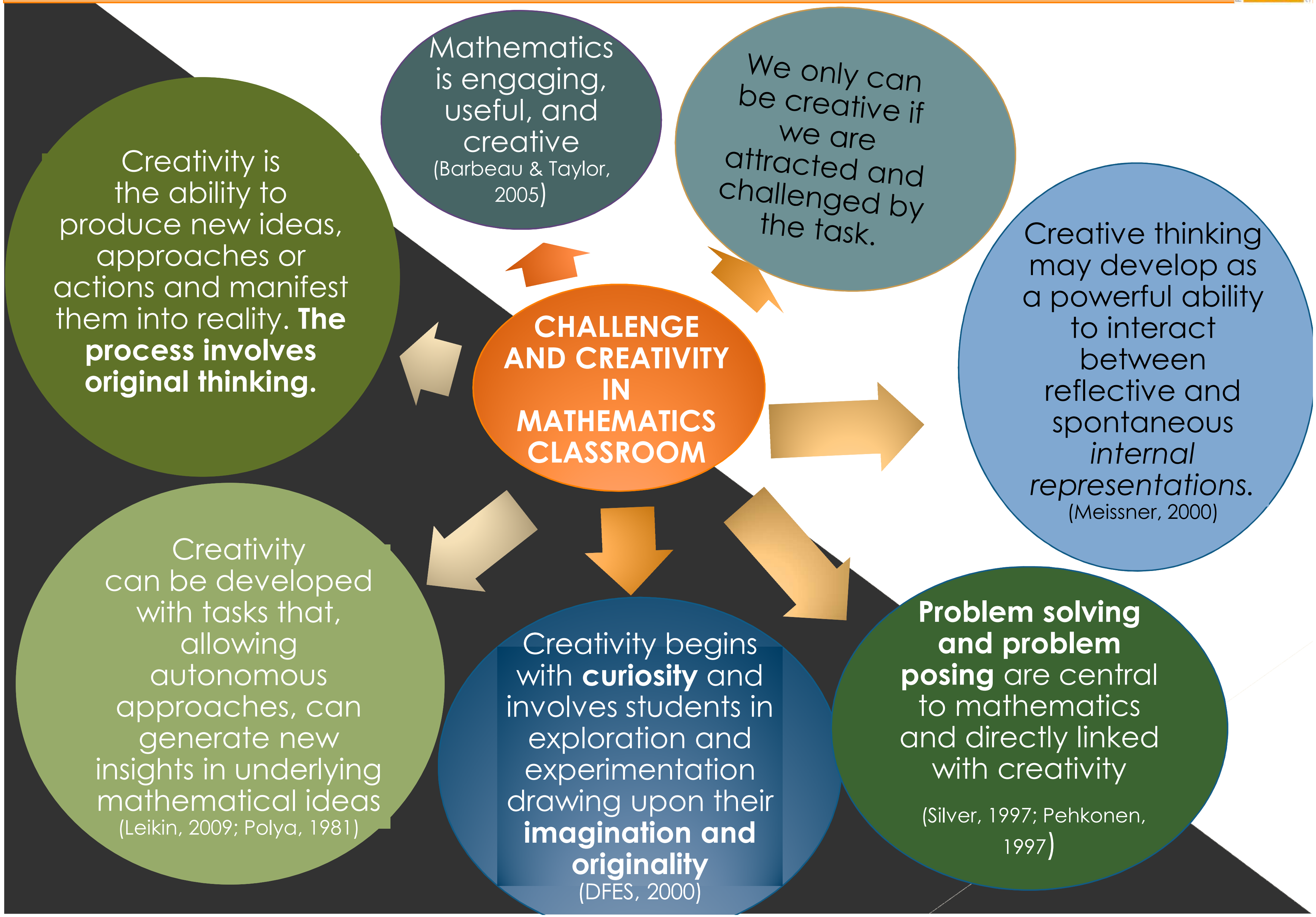
Theoretical framework

Patterning tasks allow a depth and variety of connections with all topics of mathematics, both to prepare students for further learning and to develop skills of problem solving and communication, since they

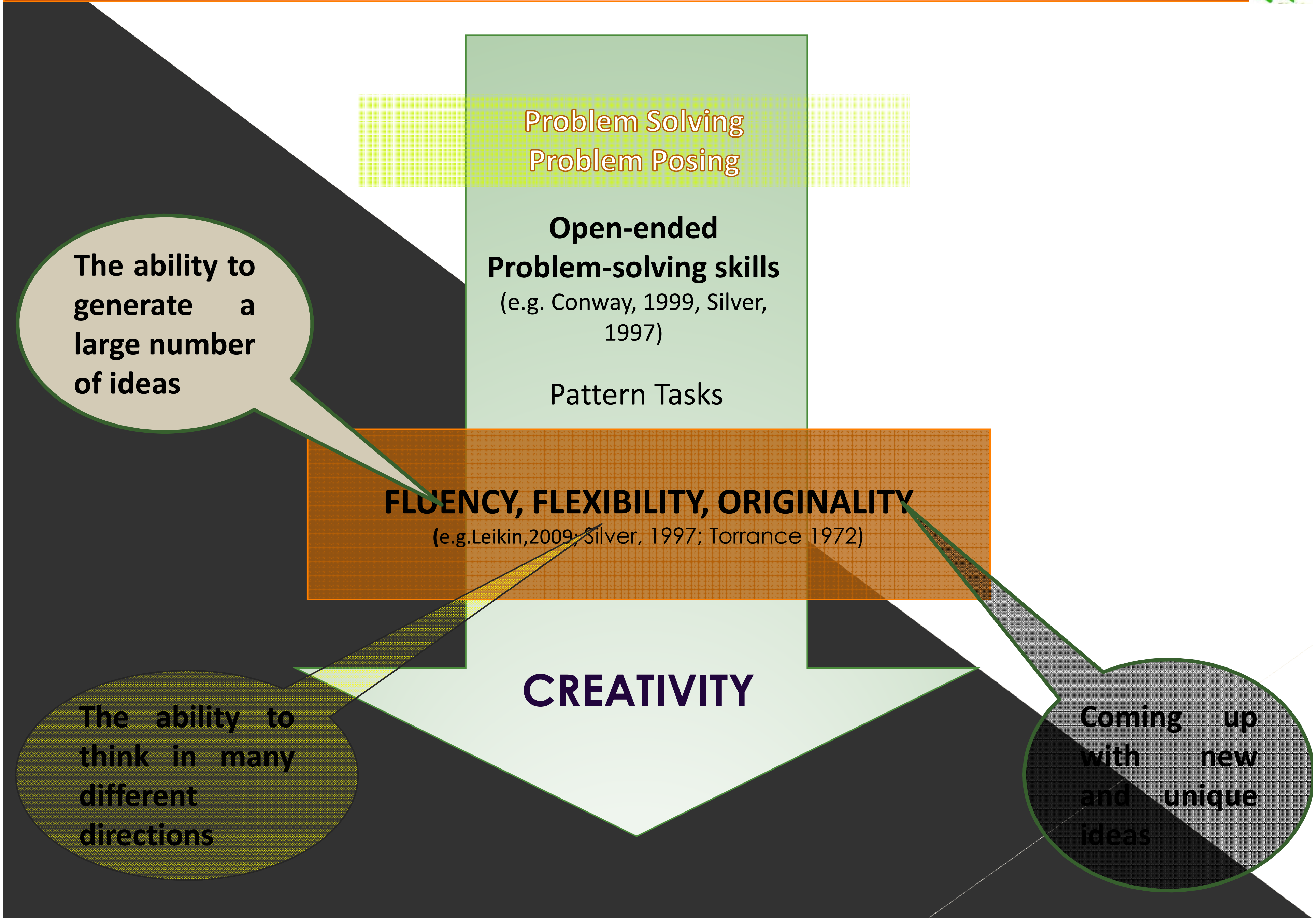
- challenge for creative ways to reach the solutions.

(NCTM, 2000; Orton, 1999; Polya, 1945; Vale et al., 2009)

Theoretical framework



Theoretical framework



Theoretical framework

Teachers with a solid knowledge on the mathematics to teach.

Mathematically rich and challenging tasks.

Exploratory teaching

Creative teachers, able to select good tasks for the use in mathematics class.

Questioning in order to provoke discussion of ideas and reflection.

Tasks that have or make use of visual / figurative contexts

Methodology

- A **qualitative exploratory** approach with elementary pre-service teachers.

- To understand in what way a didactical experience through **challenging tasks**, grounded on **figural pattern problems**, is a suitable context for promoting creativity in students solutions, in particular in getting **creative ways of expression of generalization.**

- This proposal emphasizes the **figurative contexts of patterning** as a way to **reach generalization**, through meaningful representations, in particular **the algebraic (or numeric) expressions**.

Methodology

Two main questions

- ✓ Did the pattern tasks in figurative contexts promote multiple solutions?
- ✓ How can we characterize creativity when students solve challenging pattern tasks in figurative contexts?

- The **participants** were twenty-one elementary mathematics pre-service teachers during the didactics of mathematics classes of the 3rd academic year.

- Data collected in a **holistic, descriptive and interpretive** way including classroom **observations, notes and documents** (e.g. worksheets, individual works).

- The pattern didactical proposal includes a sequence of tasks: **counting, repeating and growing patterns, and pattern problems.**

Methodology

- Data analysis includes measuring students' creativity through the three dimensions - fluency, flexibility, originality (Silver, 1997; Conway,1999):

Fluency
Fluency can be measured by the number of correct responses, solutions, obtained by the student to the same task

Flexibility
Flexibility can be measured with the number of different solutions that the student can produce organized in different categories problem.

Originality
Originality can be measured analyzing the number of responses in comparison with the percentage of students in the same group that could produce the same solution.

In this exploratory study we analyzed them **globally** identifying: the **most common** and the **most original** according to the frequency of the responses.

Results and discussion

Task 1 - The shells

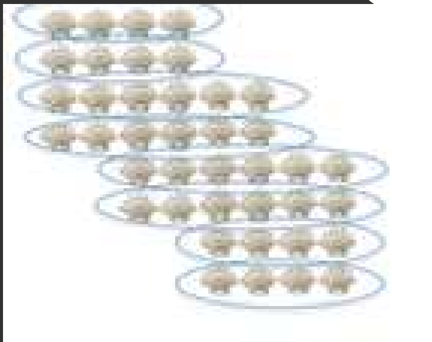
The sea girl organized the shells
she caught yesterday like the figure shows.
Can you find a quick process to count them?
Discover as much ways as you can.

This type of task requires students to see the arrangement in different ways connecting previous knowledge about numbers relationships and their connections with basic geometric concepts.

There are different ways to count the arrangement of the shells and each counting can be respectively written through a numerical expression that translates the students' thinking and seeing.

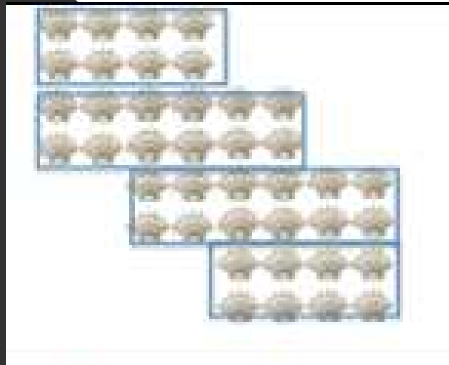
Results and discussion

The most common solutions



4+4+6+6+6+6+4+4
(20)

"I see the shells in horizontal rows each one with 4, 4, 6, 6, 6, 4 and 4 shells".

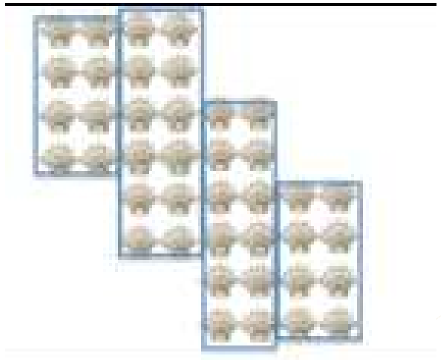


2x4+2x6+2x6+2x4
(16)

"One rectangle of 2 by 4, another rectangle of 2 by 6, another of 2 by 6 and a last one of 2 by 4".

10 x (2x2)
(10)

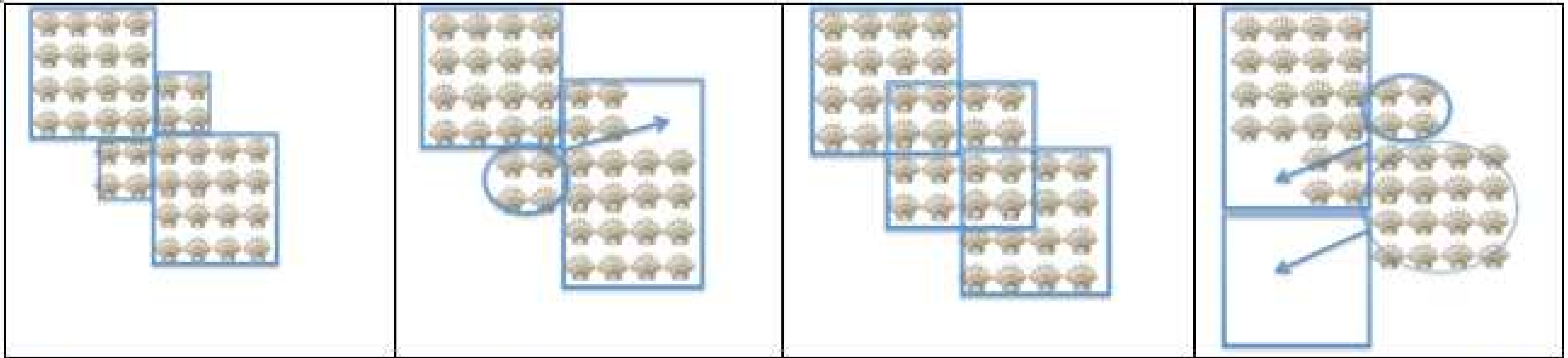
"I see ten squares of 2 by 2".



4x2+6x2+6x2+4x2
(9)

Results and discussion

The most original solutions



(4X4)+4+4+ (4X4) (4)	4x4+6x4 (2)	3x(4x4) – (4+4) (2)	10 x 4 (1)

These are the most original solutions because of the statistical infrequency of responses in relation to peer group of responses.

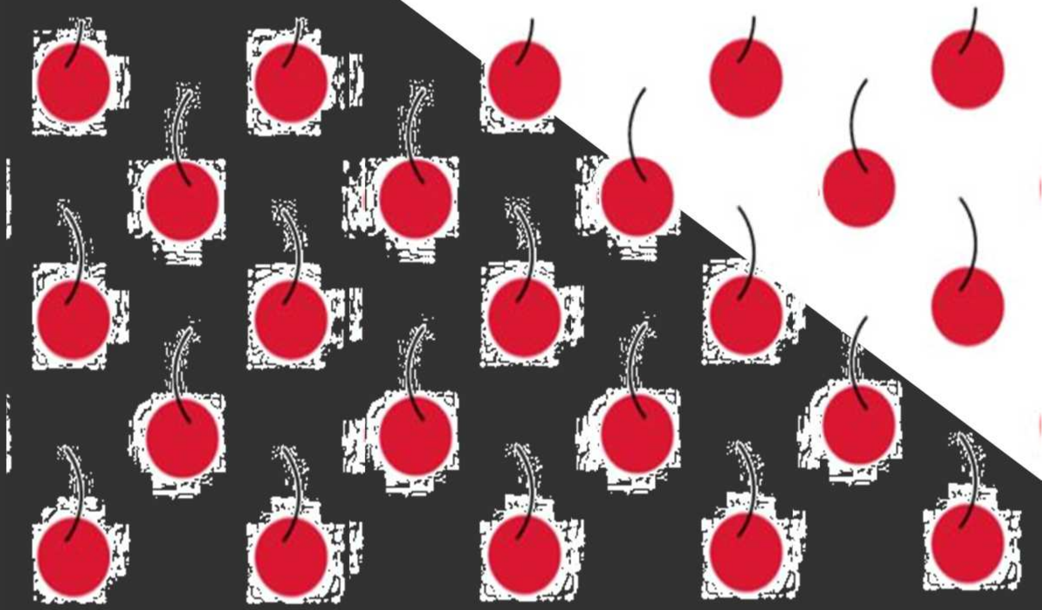
deconstructive reasoning (Rivera, 2009)

Results and discussion

Task 1 - The Cherries

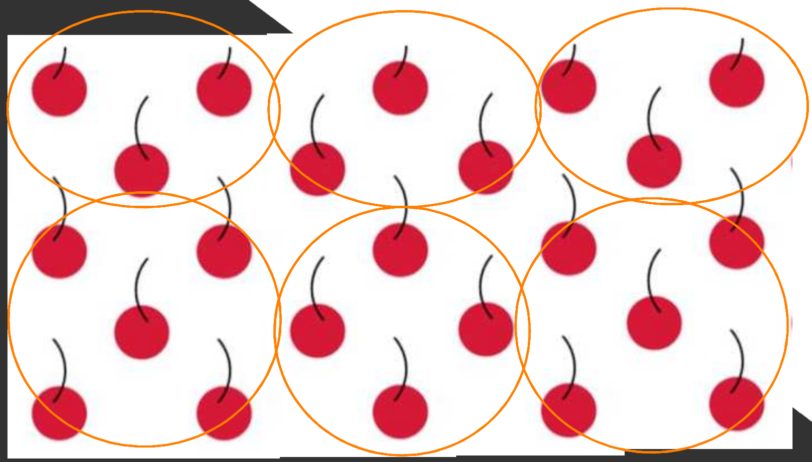
Find different ways to count the cherries.

Write the numerical expression that translate that way of counting them.

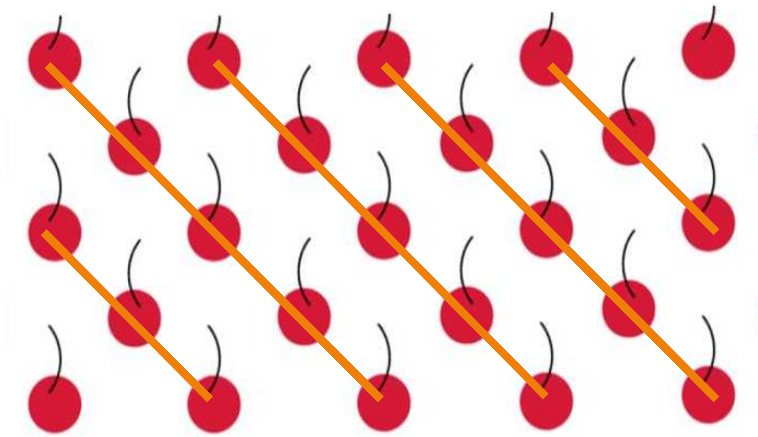


Results and discussion

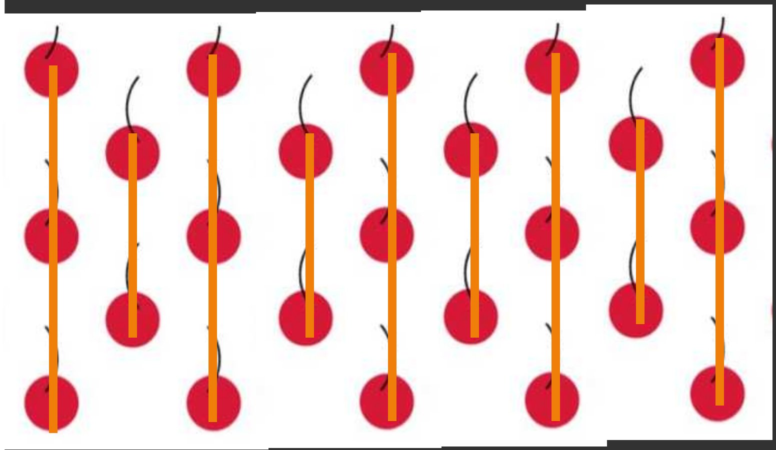
The most common solutions



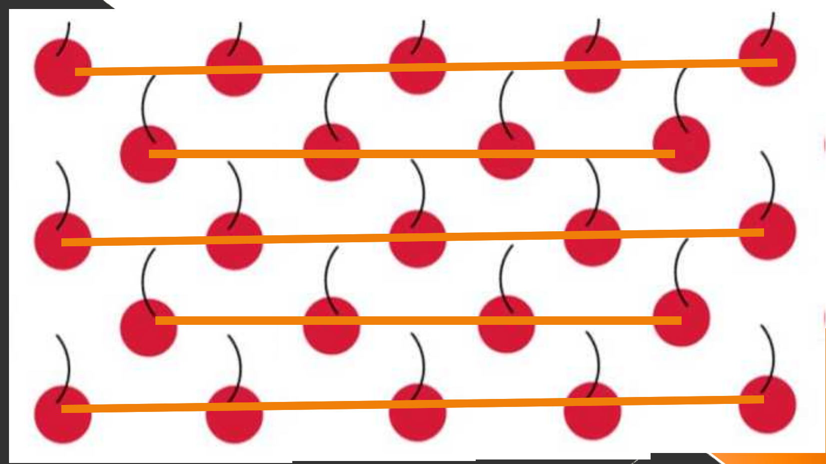
3 X 3 + 2 X 5 + 4



2 + 2 X 3 + 3 X 5



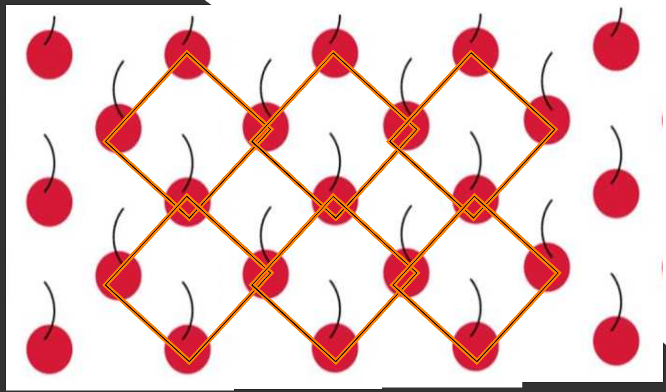
5 X 3 + 4 X 2



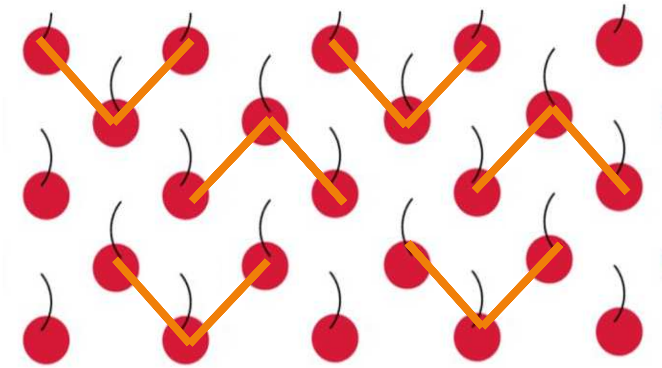
3 X 5 + 2 X 4

Results and discussion

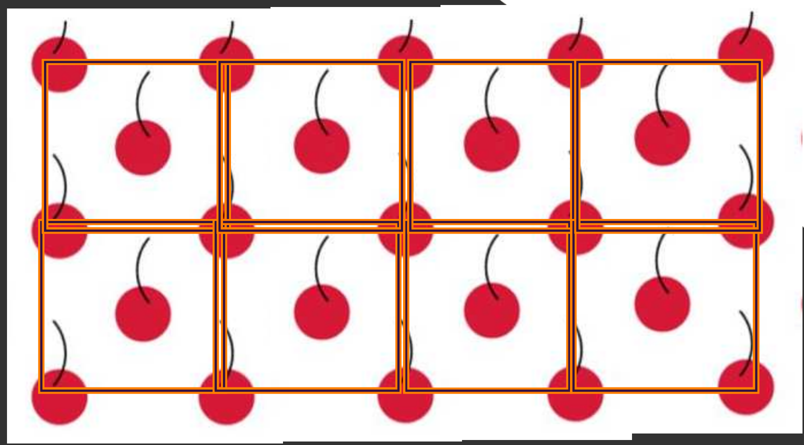
The most original solutions



2 X 3 + 6 X 4 – 7 (2)

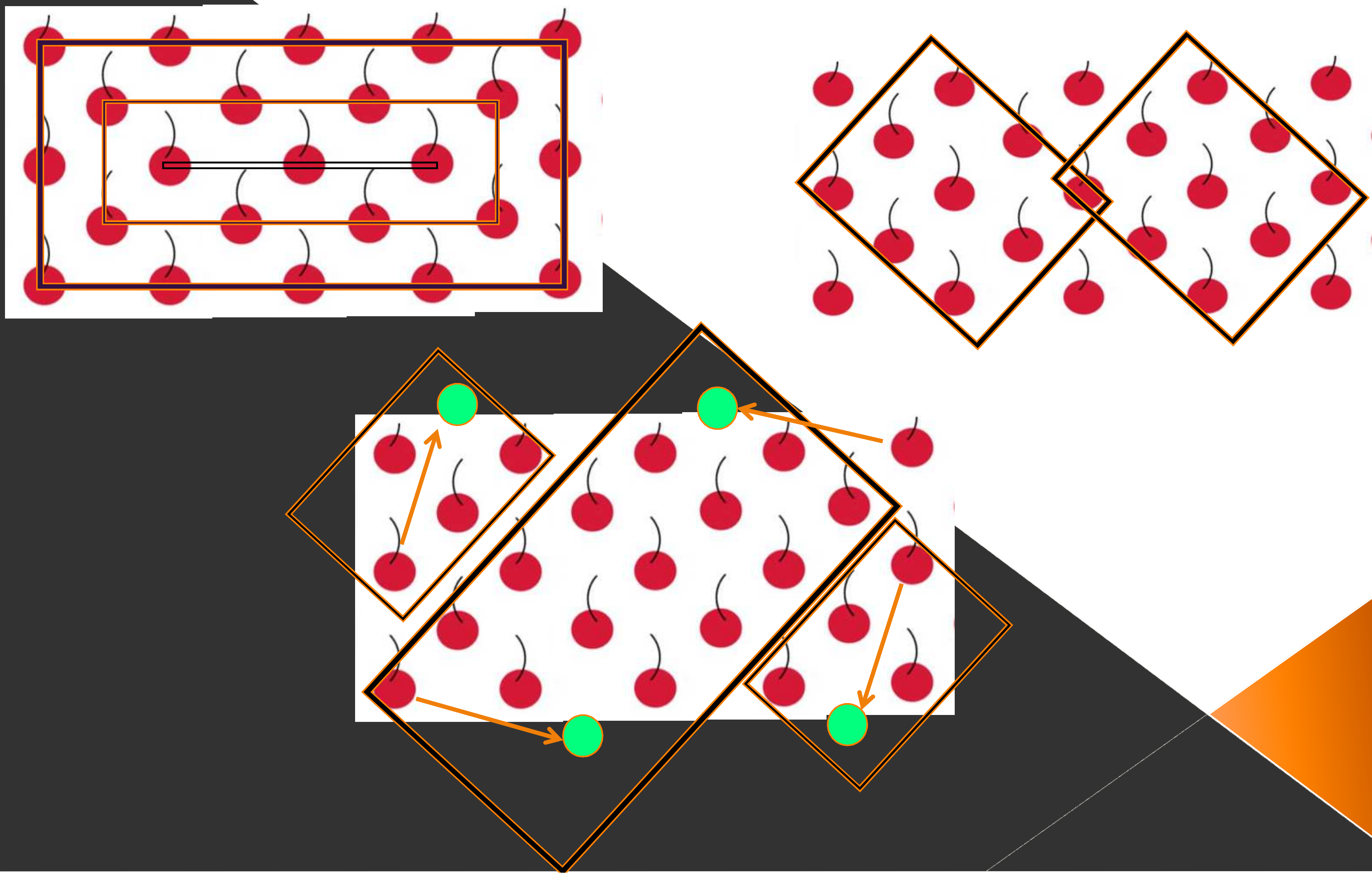


2 + 1 + 2 + 6 X 3 (1)



2 X 4 + 8 X 4 – 17 (1)

Results and discussion

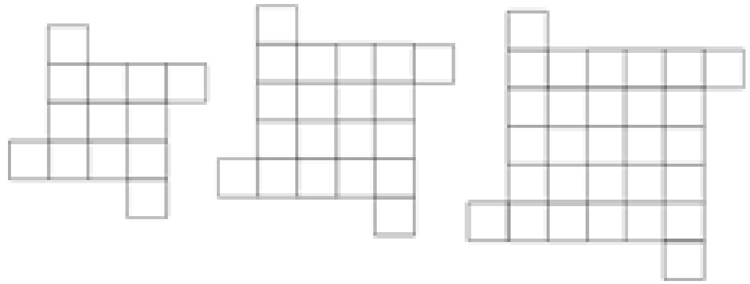


Results and discussion

Task 2 - Squares

Observe the growing pattern.

1. Draw the next figure.
2. Write the expression of the n^{th} term.



Students must look for a pattern in a figurative sequence, describe it, and produce arguments to validate it using different representations.

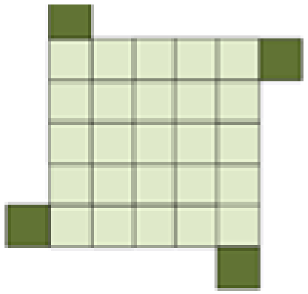
The previous work with visual counting may help to see a visual arrangement that changes in a predictable form and write numerical expressions translating the way of seeing, in order to make possible the generalization to distant terms. (Vale & Pimentel, 2011)

Students use different representations, more or less formal, to solve this task. They achieve a general rule through schemes and drawings or tables, but mainly using functional reasoning that allowed them to accomplish far generalization.

Results and discussion

We will regard only to the different ways of seeing the pattern to get far generalization, as we are convinced that it is the most important aspect of solving these tasks in which students can be creative.

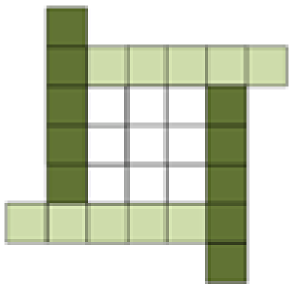
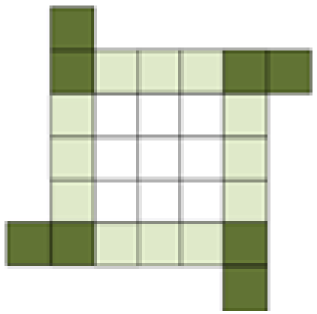
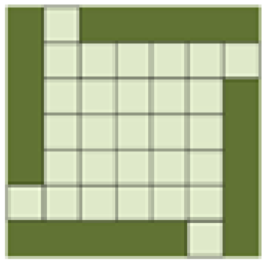
The most common solution



$(n+2)x(n+2)+4$

(10)

The most original solutions

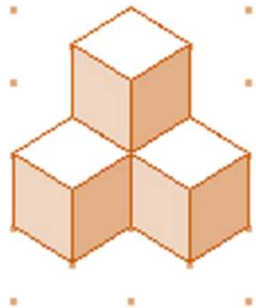
		
$nxn+4x(n+2)$ (3)	$nxn+4x2+4xn$ (3)	$(n+4)^2-4x(n+2)$ (1)

deconstructive reasoning (Rivera, 2009)

Task 4 - Cubes

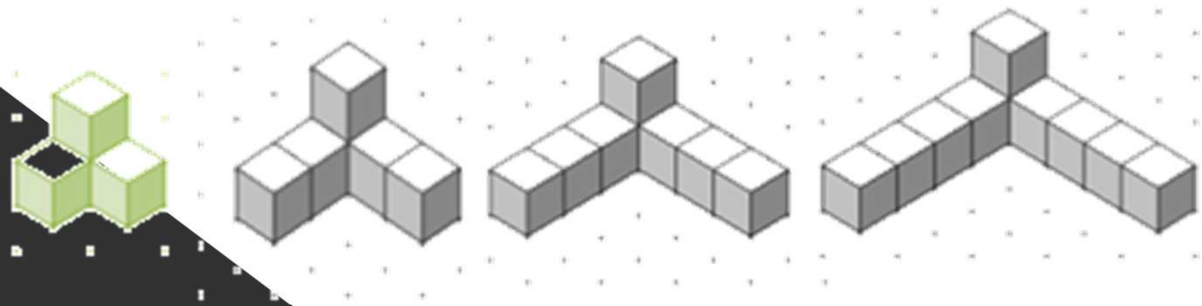
Observe the figure constructed with cubes.

1. Imagine that this figure is the 1st term of a sequence. Draw the next terms.
2. Write a numerical expression translating a way to calculate the nth number term of the sequence you built.
3. Construct as much sequences as you can.

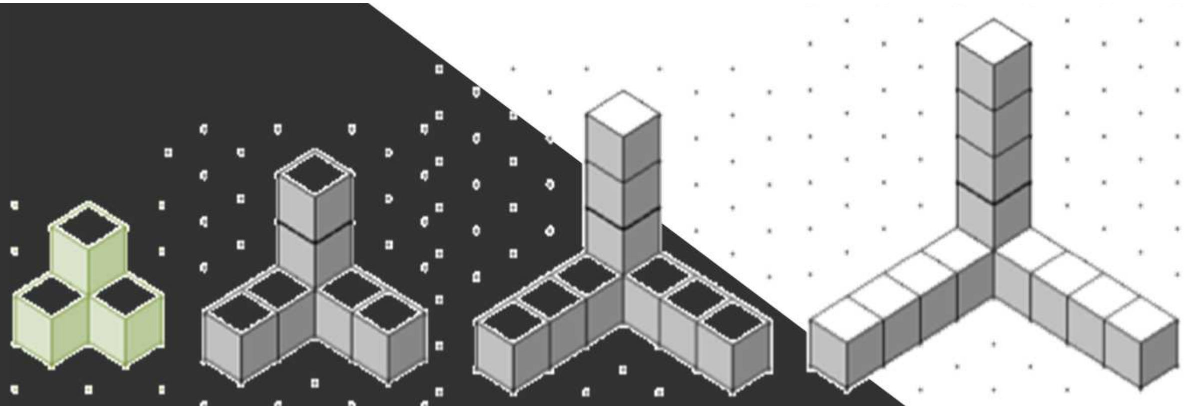


Results and discussion

The most common solutions



2+2xn 17

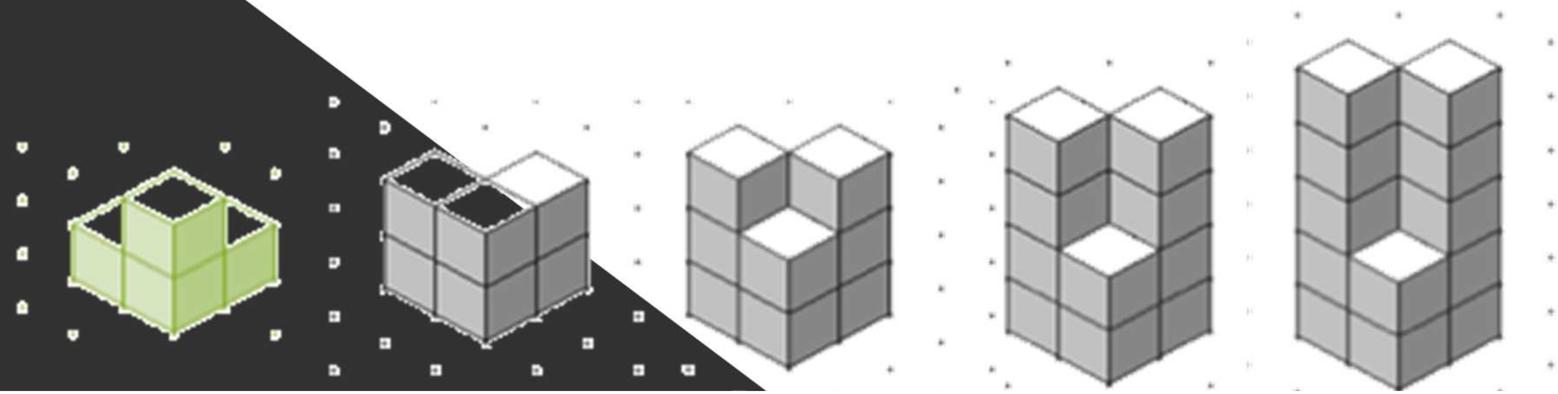


1+3xn 15

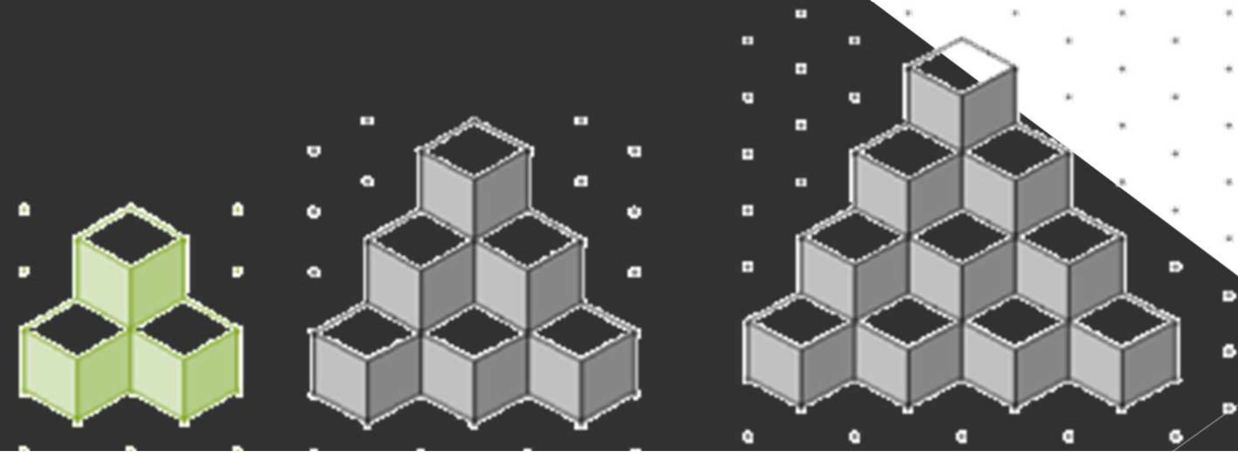
3+n 9

Results and discussion

The most original solutions



2+2n) 1



n (n+1) (n+2)/6 1

Concluding remarks

- Students need to recognize that both flexibility and originality encourage divergent thinking, which promotes higher-level thinking.

- The challenge is to provide an environment of practice and problem solving that stimulates creativity that will enable the development of mathematical competence in all students.

- Our concern was not to categorize students but to identify potentialities in the tasks to develop creativity in students, detecting their mathematical strengths or weaknesses.

Concluding remarks

- Fluency and flexibility were largely identified mainly in the counting tasks.

- We must look for ways to improve originality that in this class had not high results.

- Future teachers must become themselves creative thinkers and they must be aware to act in the same way with their own students, encouraging them to seek unusual and original responses.

- Creativity is a field that we are just beginning to explore but this allowed us to experience the construction of some tasks that, in addition to the mathematical concepts and processes they involve, mainly generalization, allow students get multiple solutions.

Creative … oking
at the sa… e else
and think… ent.

Albert Szent-Gyorgi, Nobel Prize of Medicine 1937

Theoretical framework

Visual counting tasks in different contexts

Patterns recognition in different arranges to develop subitizing and to facilitate counting (e.g. association to get fives or tens, multiplication in rectangular arrays)

Sequence tasks

To look for patterns in sequences - repeating and growing patterns - in order to get generalization (near and far generalization) through rules that students can formulate and allow students to get recursive and functional reasoning.

Problem tasks

In these tasks students may have to construct their own sequences and/or to discover the pattern to reach generalization in order to get the solution.

Generalization:

mathematical concepts and properties. Algebraic thinking.

The didactical experience

introduction

- Finding creative students is a challenge in today's society and consequently in school. This implies changes in classroom practices and curriculum materials.

- To construct a mathematics curriculum that gives students opportunities to be imaginative and make mathematics with sense, playful, challenging and useful in today's world.